Today’s agenda - part I:

**Magnetic Flux and Gauss’ Law for Magnetism.**
You must be able to calculate magnetic flux and recognize the consequences of Gauss’ Law for Magnetism.

Magnetic Fields Due To A Moving Charged Particle.
You must be able to calculate the magnetic field due to a moving charged particle.

Biot-Savart Law: Magnetic Field due to a Current Element.
You must be able to use the Biot-Savart Law to calculate the magnetic field of a current-carrying conductor (for example: a long straight wire).

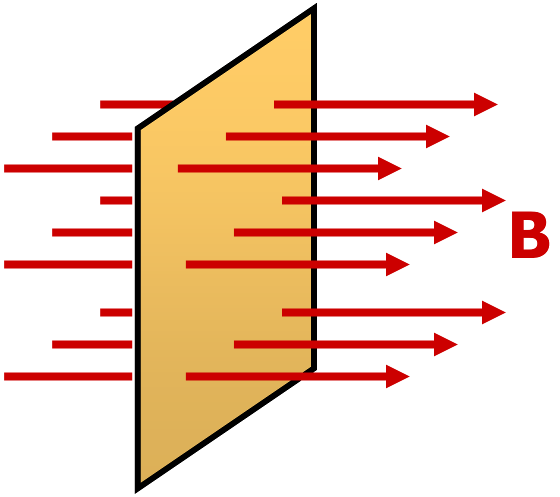
Force Between Current-Carrying Conductors.
You must be able to calculate forces between current-carrying conductors

*last week we studied the effects of magnetic fields on charges and wires with current, today we learn how to produce magnetic fields

# Magnetic Flux and Gauss' Law for Magnetism

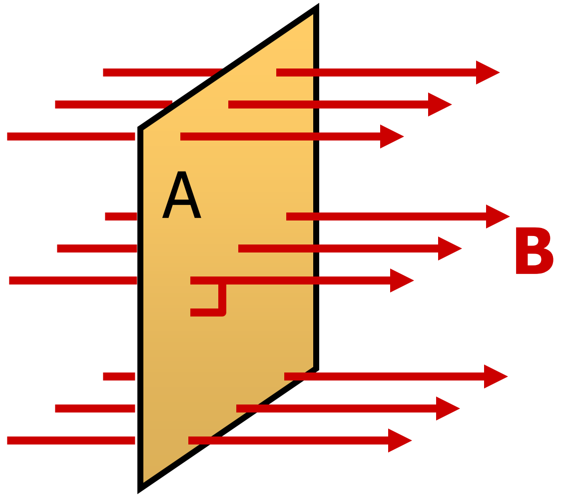
**Define magnetic flux:**

- <u>in complete analogy to electric flux</u>
- count number of magnetic field lines passing through a surface

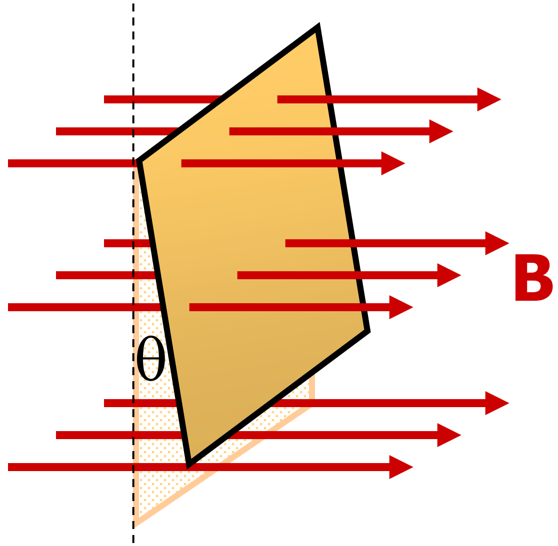


magnetic flux passing through a surface is (proportional to) number of magnetic field lines that pass through it

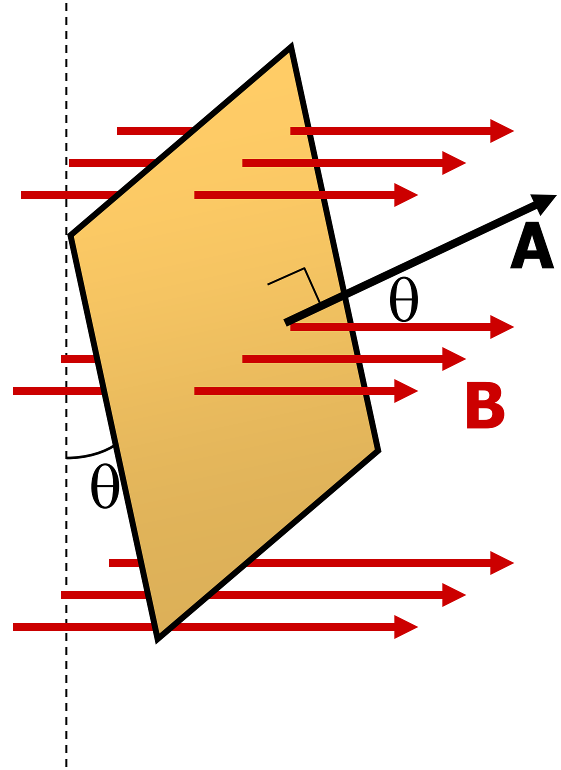
if B is uniform and normal to surface
$\Phi_B = BA$.




if the surface is tilted, fewer lines cut the surface.



$\vec{A}$ is vector having a magnitude equal to surface area, in direction normal to surface.



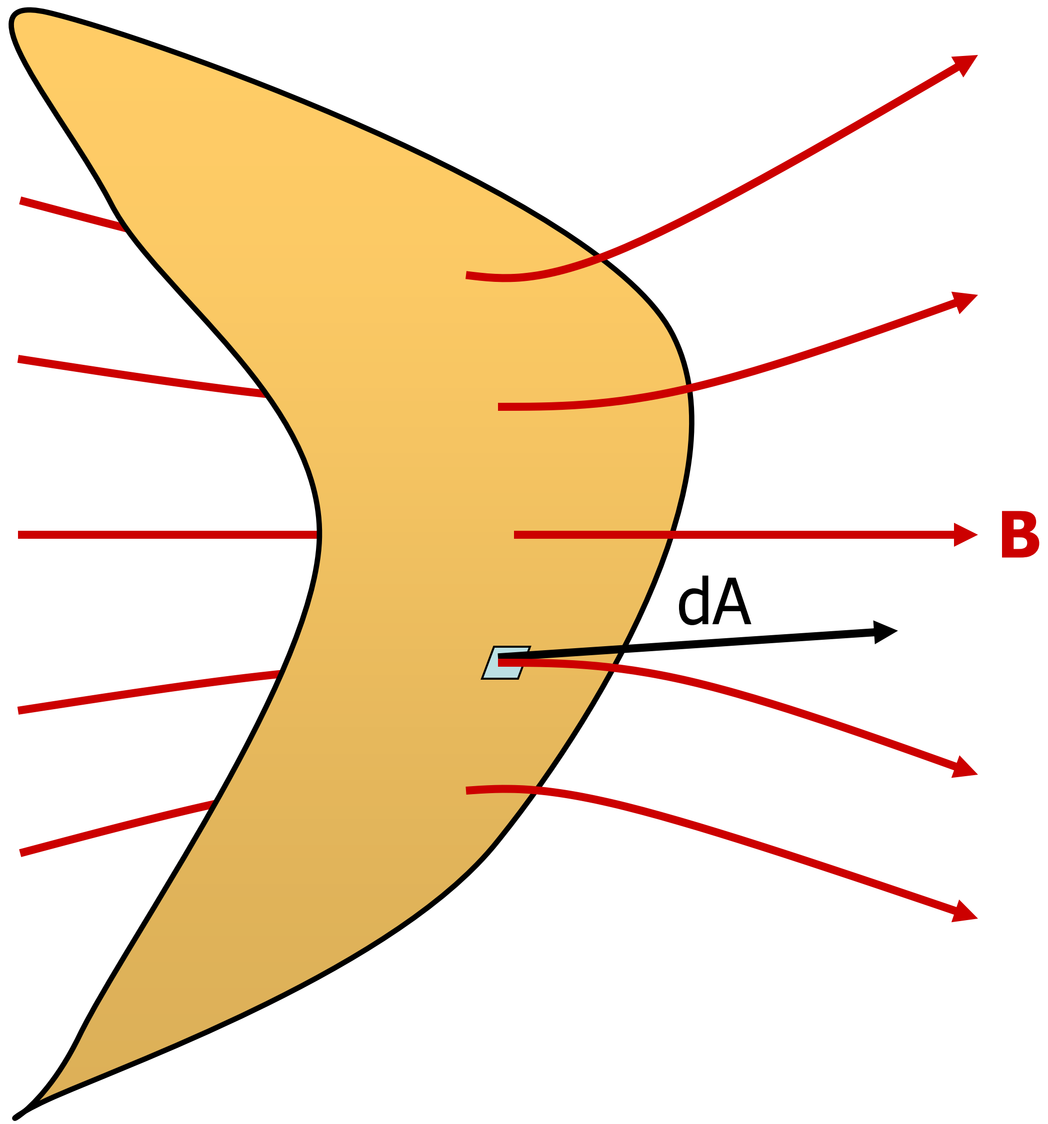

The "amount of surface" perpendicular to magnetic field is $A\cos\theta$.

Because $\vec{A}$ is perpendicular to surface, amount of $\vec{A}$ parallel to magnetic field is $A\cos\theta$.

$$A_{\parallel} = A\cos\theta \quad \text{so} \quad \Phi_B = BA_{\parallel} = BA\cos\theta.$$

$$\Phi_B = \vec{B}\cdot\vec{A}$$

# If magnetic field is not uniform, or surface is not flat…




divide surface into infinitesimal surface elements and add flux through each…

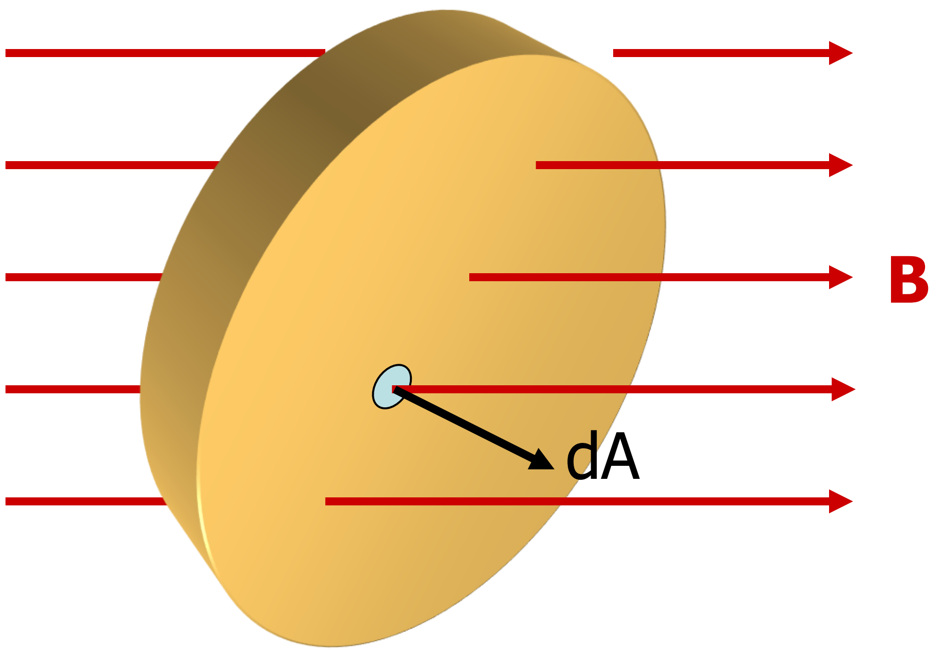
$$\Phi_B = \lim_{\Delta A_i \to 0} \sum_i \vec{B}_i \cdot \Delta\vec{A}_i$$

$$\boxed{\Phi_B = \int \vec{B} \cdot d\vec{A}}$$

**definition of magnetic flux**

(similar to the definition of electric flux)

If the surface is closed (completely encloses a volume)…



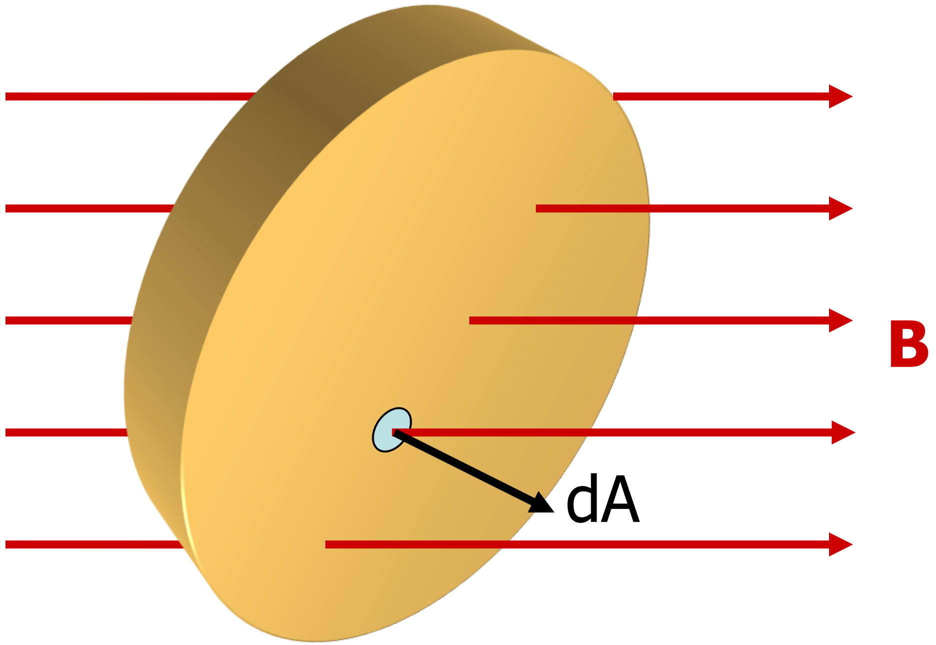

…we count lines going out as positive and lines going in as negative…

$$\Phi_B = \oint \vec{B} \cdot d\vec{A}$$

a surface integral, therefore a double integral $\iint$

**Recall:**

- field lines begin and end at charges (monopoles)
- there are **no magnetic monopoles** in nature
- **all field lines entering surface have to leave it again**




Therefore

$$\boxed{\Phi_M = \oint \vec{B} \cdot d\vec{A} = 0}$$

## Gauss’ Law for Magnetism!

This law may require modification if the existence of magnetic monopoles is confirmed.

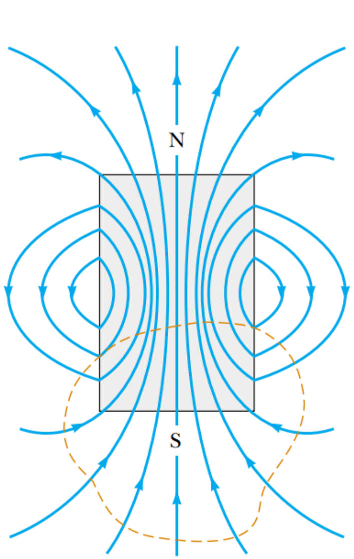
Gauss’ Law for magnetism is not very useful if we are considering stationary cases.
The concept of magnetic flux is extremely useful, and will be used later, when the magnetic flux will change over time!

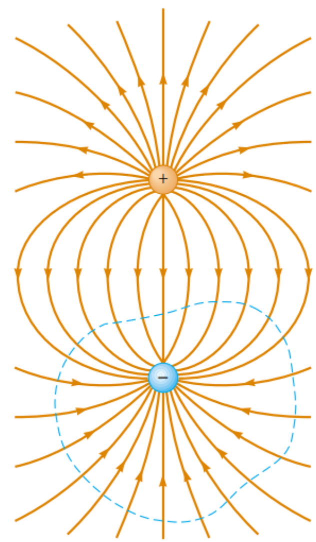
You have now learned Gauss's Law for both electricity and magnetism.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} \qquad \oint \vec{B} \cdot d\vec{A} = 0$$

These equations can also be written in <u>differential form</u>:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \qquad \vec{\nabla} \cdot \vec{B} = 0$$




The magnetic field lines of a bar magnet form closed loops. Note that the net magnetic flux through a closed surface surrounding one of the poles (or any other closed surface) is zero. (The dashed line represents the intersection of the surface with the page.)

Today's agenda - part I:

Magnetic Flux and Gauss' Law for Magnetism.

You must be able to calculate magnetic flux and recognize the consequences of Gauss' Law for Magnetism.

**Magnetic Fields Due To A Moving Charged Particle.**

You must be able to calculate the magnetic field due to a moving charged particle.

Biot-Savart Law: Magnetic Field due to a Current Element.

You must be able to use the Biot-Savart Law to calculate the magnetic field of a current-carrying conductor (for example: a long straight wire).

Force Between Current-Carrying Conductors.

You must be able to calculate forces between current-carrying conductors

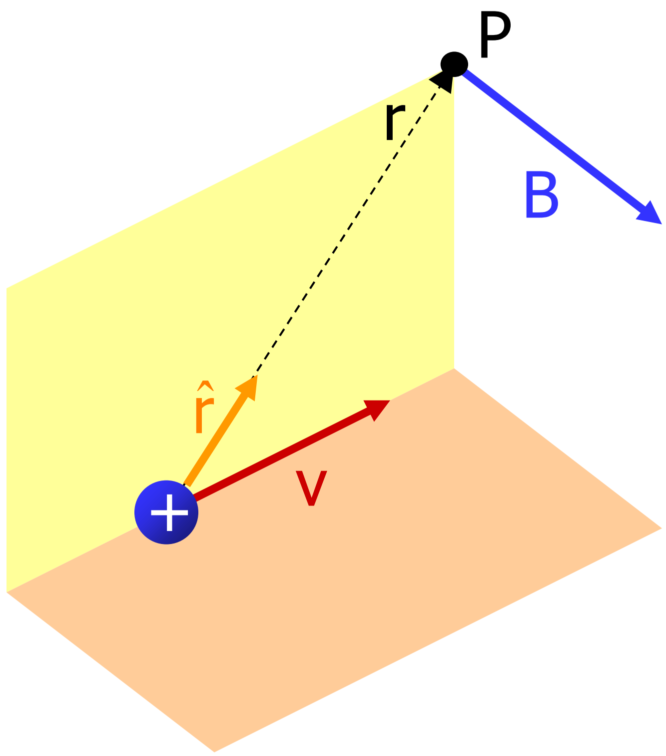
*****last week we studied the effects of magnetic fields on charges, today we learn how to produce magnetic fields

# Magnetic Field of a Moving Charged Particle

- moving charge creates magnetic field

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}.}$$

$\mu_0$ is a constant, $\mu_0 = 4\pi x 10^{-7}$ T·m/A




Remember:
$\hat{r}$ is unit vector from source point (the thing that causes the field) to the field point P (location where the field is being measured).

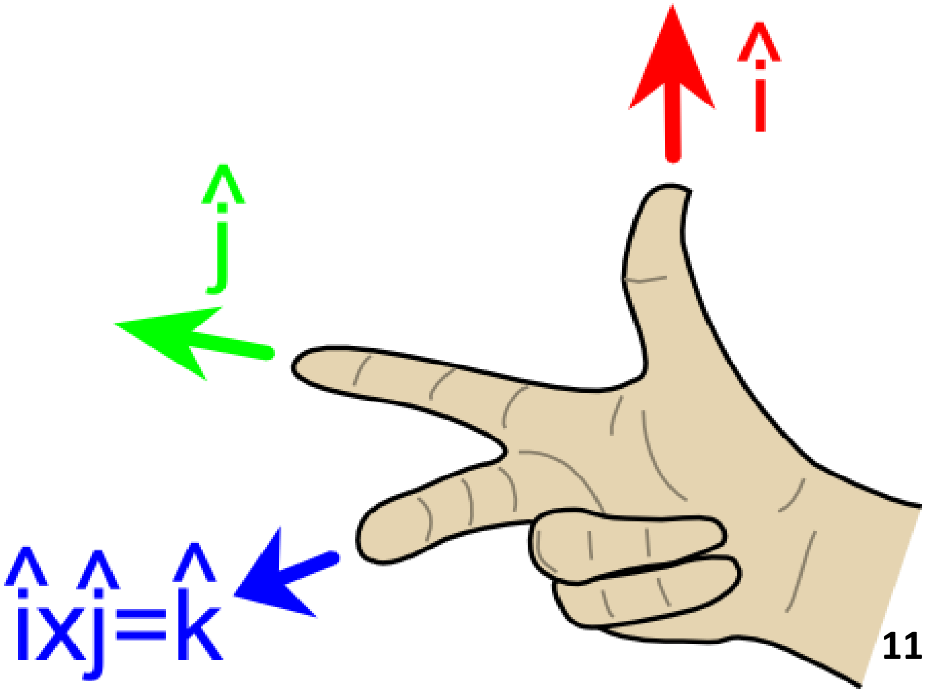
# Detour: cross products of unit vectors

- need lots of cross products of unit vectors $\hat{i}, \hat{j}, \hat{k}$

**Work out determinant:**

Example: $\hat{k} \times \left(-\hat{j}\right) = \det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} = \hat{i}\left(0-(-1)\right) = \hat{i}$

**Use right-hand rule:**



# Detour: cross products of unit vectors

## Cyclic property:

**"forward"**

i j k i j k

$$\hat{i} \times \hat{j} = \hat{k}$$

**"backward"**

i j k i j k

$$\hat{j} \times \hat{i} = -\hat{k}$$

Today's agenda - part I:

Magnetic Flux and Gauss' Law for Magnetism.

You must be able to calculate magnetic flux and recognize the consequences of Gauss' Law for Magnetism.

Magnetic Fields Due To A Moving Charged Particle.

You must be able to calculate the magnetic field due to a moving charged particle.

**Biot-Savart Law: Magnetic Field due to a Current Element.**

You must be able to use the Biot-Savart Law to calculate the magnetic field of a current-carrying conductor (for example: a long straight wire).

Force Between Current-Carrying Conductors.

You must be able to calculate forces between current-carrying conductors

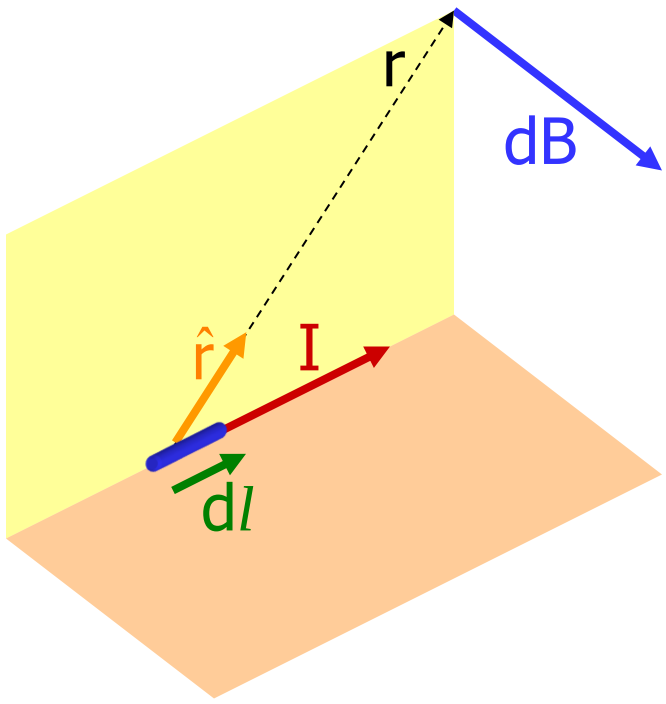
*last week we studied the effects of magnetic fields on charges, today we learn how to produce magnetic fields

# Biot-Savart Law: magnetic field of a current element

- moving charge creates magnetic field

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}.$$

- current I in infinitesimal length $d\vec{\ell}$ of wire gives rise to magnetic field $d\vec{B}$



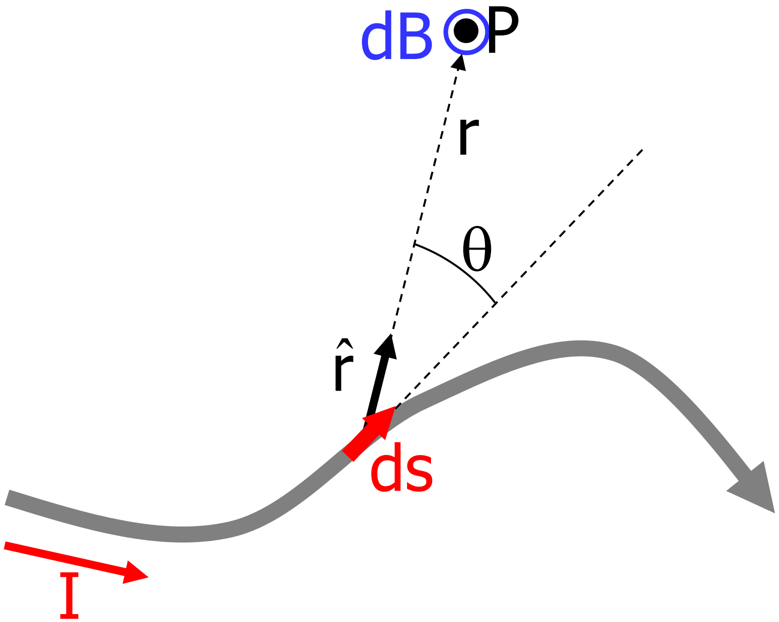

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\ell}\times\hat{r}}{r^2}$$

**Biot-Savart Law**

Derived by summing contributions of all charges in wire element

You may see the equation written using $\vec{r} = r\,\hat{r}$.

# Applying the Biot-Savart Law




$$d\vec{B} = \frac{\mu_0}{\mathbf{4\pi}} \frac{I\, d\vec{s} \times \hat{r}}{r^2}$$

$$|d\vec{s} \times \hat{r}| = |d\vec{s}|\, |\hat{r}| \sin\theta$$

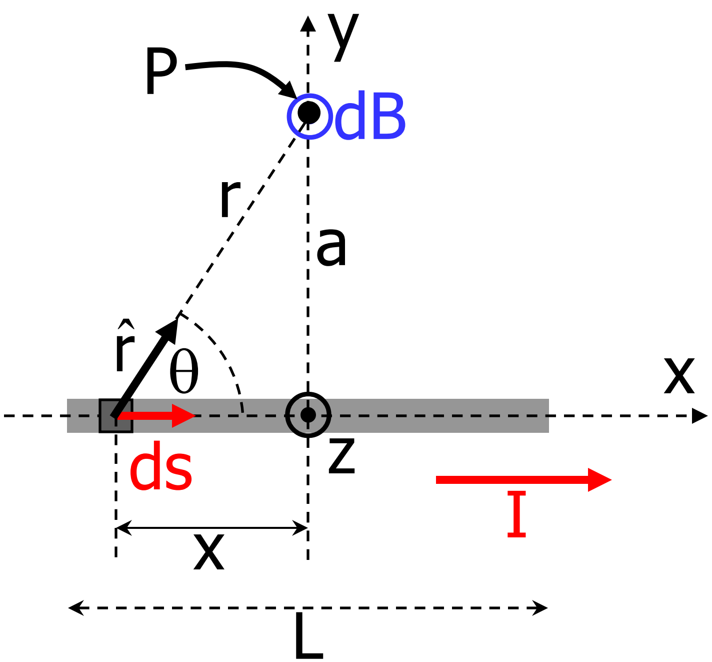
$$= ds \sin\theta \quad \text{because } |\hat{r}| = 1$$

$$dB = \frac{\mu_0}{\mathbf{4\pi}} \frac{I\, ds \sin\theta}{r^2}$$

$$\vec{B} = \int d\vec{B}$$

Hint: if you have a tiny piece of a wire, just calculate dB; no need to integrate.

Example: calculate the magnetic field at point P due to a thin straight wire of length L carrying a current I. (P is on the perpendicular bisector of the wire at distance a.)




$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{s} \times \hat{r}}{r^2}$$

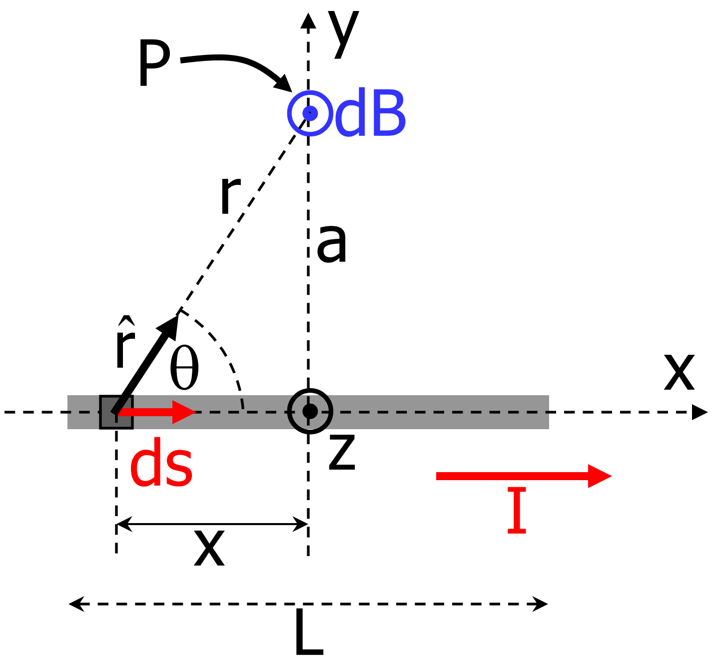
$$d\vec{s} \times \hat{r} = ds \sin\theta\, \hat{k}$$

$$dB = \frac{\mu_0}{4\pi} \frac{I\, ds \sin\theta}{r^2}$$

ds is an infinitesimal quantity in the direction of dx, so

$$dB = \frac{\mu_0}{4\pi} \frac{I\, dx \sin\theta}{r^2}$$

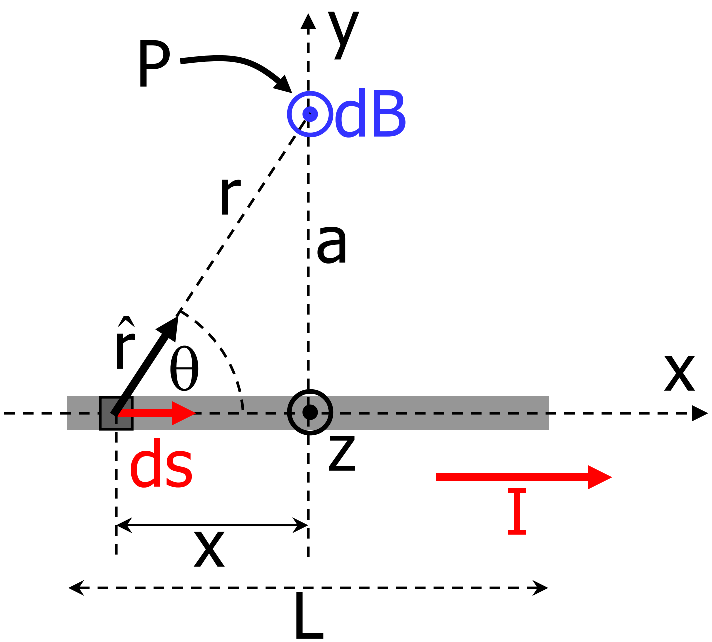
$$\sin\theta = \frac{a}{r} \qquad r = \sqrt{x^2 + a^2} \qquad dB = \frac{\mu_0}{4\pi}\frac{I\,dx\,\sin\theta}{r^2}$$




$$dB = \frac{\mu_0}{4\pi}\frac{I\,dx\,a}{r^3} = \frac{\mu_0}{4\pi}\frac{I\,dx\,a}{(x^2 + a^2)^{3/2}}$$

$$B = \int_{-L/2}^{L/2} \frac{\mu_0}{4\pi}\frac{I\,dx\,a}{(x^2 + a^2)^{3/2}}$$

$$B = \frac{\mu_0 I\,a}{4\pi}\int_{-L/2}^{L/2} \frac{dx}{(x^2 + a^2)^{3/2}}$$



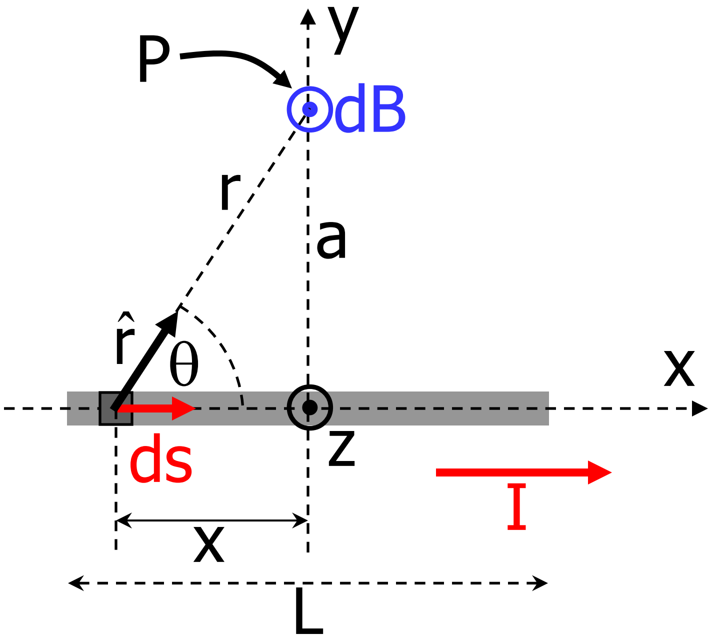

$$B = \frac{\mu_0 I\, a}{4\pi}\int_{-L/2}^{L/2} \frac{dx}{(x^2 + a^2)^{3/2}}$$

look integral up in tables, use the web,or use trig substitutions

$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2(x^2 + a^2)^{1/2}}$$

$$B = \frac{\mu_0 I\, a}{4\pi} \frac{x}{a^2(x^2 + a^2)^{1/2}}\Bigg|_{-L/2}^{L/2}$$

$$= \frac{\mu_0 I\, a}{4\pi}\left[\frac{L/2}{a^2\left((L/2)^2 + a^2\right)^{1/2}} - \frac{-L/2}{a^2\left((-L/2)^2 + a^2\right)^{1/2}}\right]$$

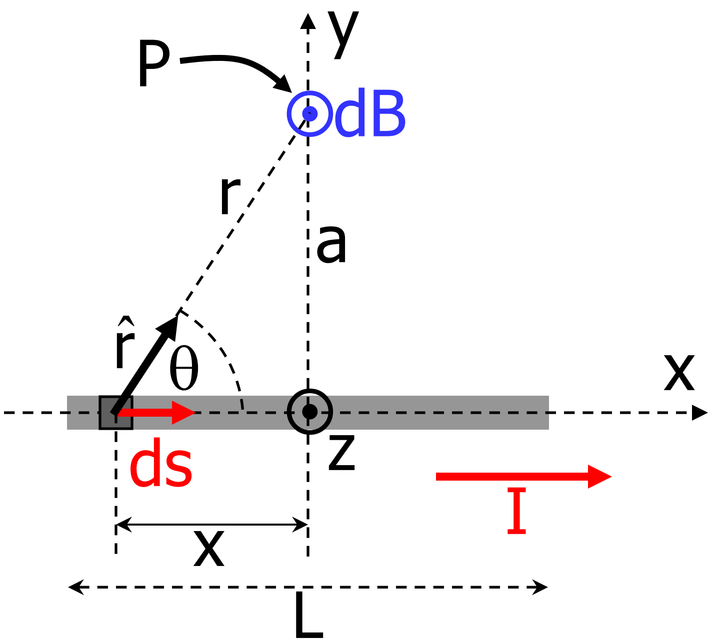



$$B = \frac{\mu_0 I\, a}{\mathbf{4\pi}}\left[\frac{2L/2}{a^2(L^2/4 + a^2)^{1/2}}\right]$$

$$B = \frac{\mu_0 I\, L}{\mathbf{4\pi}a}\frac{1}{(L^2/4 + a^2)^{1/2}}$$

$$B = \frac{\mu_0 I\, L}{2\pi a}\frac{1}{\sqrt{L^2 + 4a^2}}$$

$$B = \frac{\mu_0 I}{2\pi a}\frac{1}{\sqrt{1 + \frac{4a^2}{L^2}}}$$




$$B = \frac{\mu_0 I}{2\pi a} \frac{1}{\sqrt{1 + \frac{4a^2}{L^2}}}$$

When L→∞, $B = \frac{\mu_0 I}{2\pi a}$.

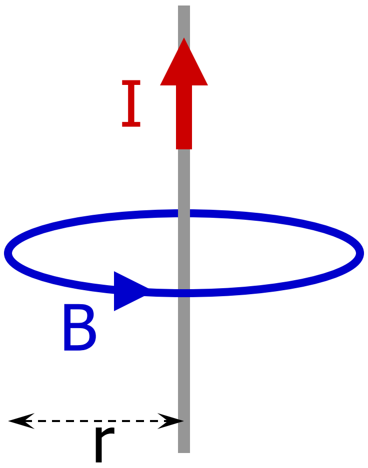
magnetic field around a long, straight wire

# Magnetic Field of a Long Straight Wire

It is possible to derived the equation for the magnetic field around a long, straight* wire using Biot-Savart law…(see previous slides)

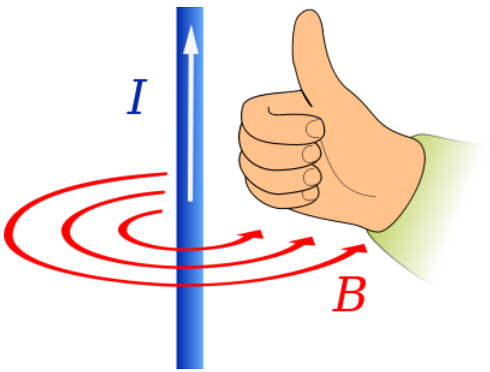
$$\boxed{B = \frac{\mu_0 \, I}{2\pi r}}$$

r is shortest (perpendicular) distance between field point and wire



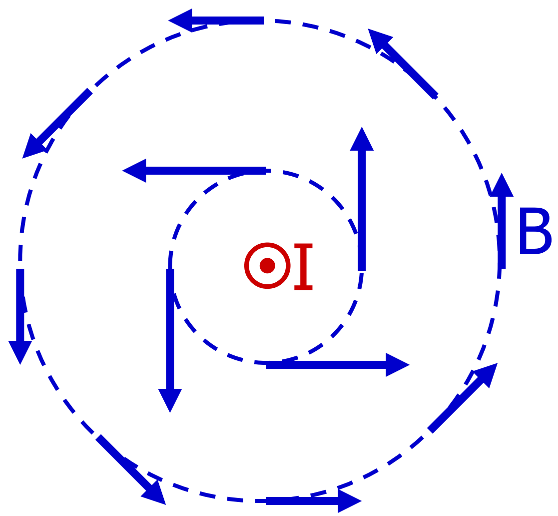

We will find soon this relation making use of the **Ampere's Law**!

…with a direction given by a "new" right-hand rule.




*Don't use this equation unless you have a long, straight wire!

Looking “down” along the wire:




- magnetic field is not constant

- at fixed distance r from wire, **magnitude** of field is constant (but vector magnetic field is not uniform).

- magnetic field **direction** is a tangent to imaginary circles around wire

Today's agenda - part I:

## Magnetic Flux and Gauss' Law for Magnetism.

You must be able to calculate magnetic flux and recognize the consequences of Gauss' Law for Magnetism.

## Magnetic Fields Due To A Moving Charged Particle.

You must be able to calculate the magnetic field due to a moving charged particle.

## Biot-Savart Law: Magnetic Field due to a Current Element.

You must be able to use the Biot-Savart Law to calculate the magnetic field of a current-carrying conductor (for example: a long straight wire).

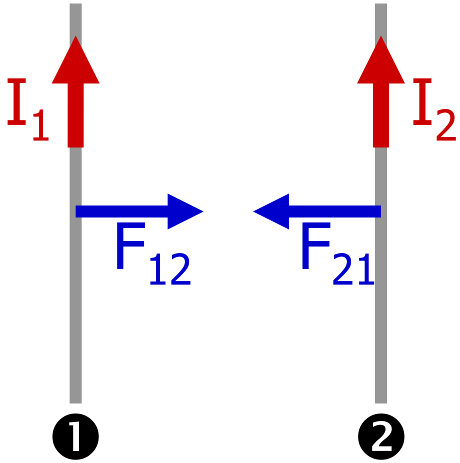
## **Force Between Current-Carrying Conductors.**

You must be able to calculate forces between current-carrying conductors

*****last week we studied the effects of magnetic fields on charges, today we learn how to produce magnetic fields

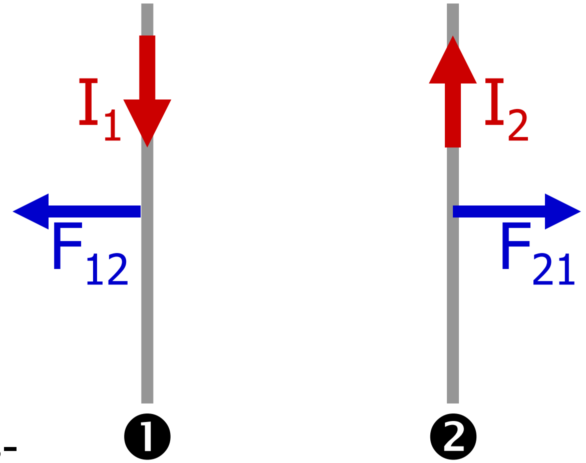
# Magnetic Field of a Current-Carrying Wire

It is experimentally observed that parallel wires exert forces on each other when current flows.




***DEMO***

https://auditoires-physique.epfl.ch/experiment/592/force-entre-deux-conducteurs-paralleles-exp-dampere



## Example: use the expression for B due to a current-carrying wire to calculate the force between two current-carrying wires.

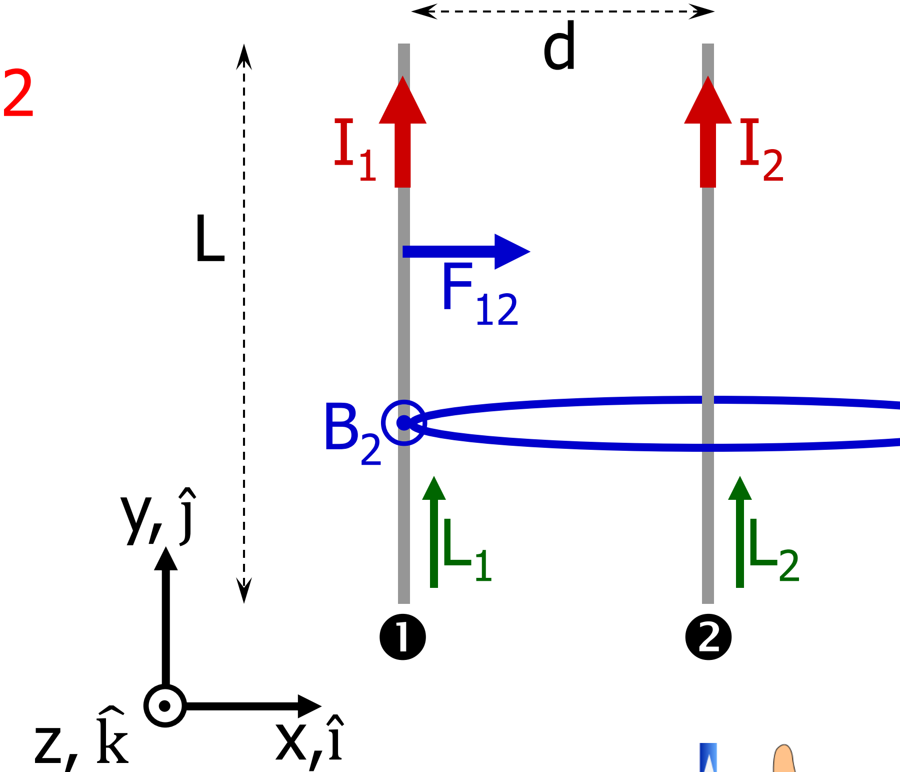
### Force on wire 1 produced by wire 2

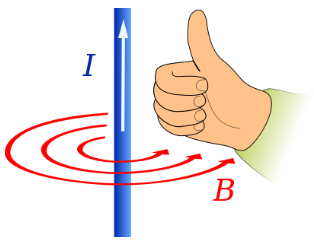
$$\vec{F}_{12} = I_1 \vec{L}_1 \times \vec{B}_2$$

$$\vec{B}_2 = \frac{\mu_0 I_2}{2\pi d} \hat{k}$$

$$\vec{F}_{12} = I_1 L \hat{j} \times \frac{\mu_0 I_2}{2\pi d} \hat{k}$$

$$\vec{F}_{12} = \frac{\mu_0 I_1 I_2 L}{2\pi d} \hat{\imath}$$




force per unit length of wire i: $\frac{\vec{F}_{12}}{L} = \frac{\mu_0 I_1 I_2}{2\pi d} \hat{\imath}.$

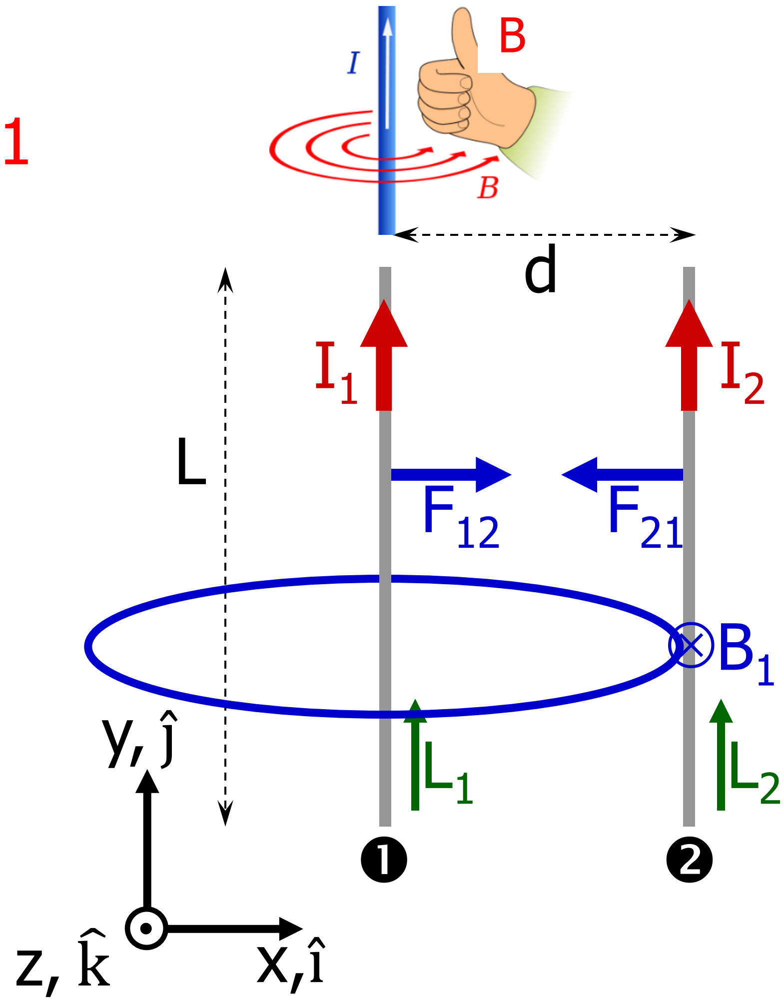
# Force on wire 2 produced by wire 1

$$\vec{F}_{21} = I_2 \vec{L}_2 \times \vec{B}_1$$

$$\vec{B}_1 = -\frac{\mu_0 I_1}{2\pi d}\hat{k}$$

$$\vec{F}_{21} = I_2 L\hat{j} \times \left(-\frac{\mu_0 I_1}{2\pi d}\hat{k}\right)$$

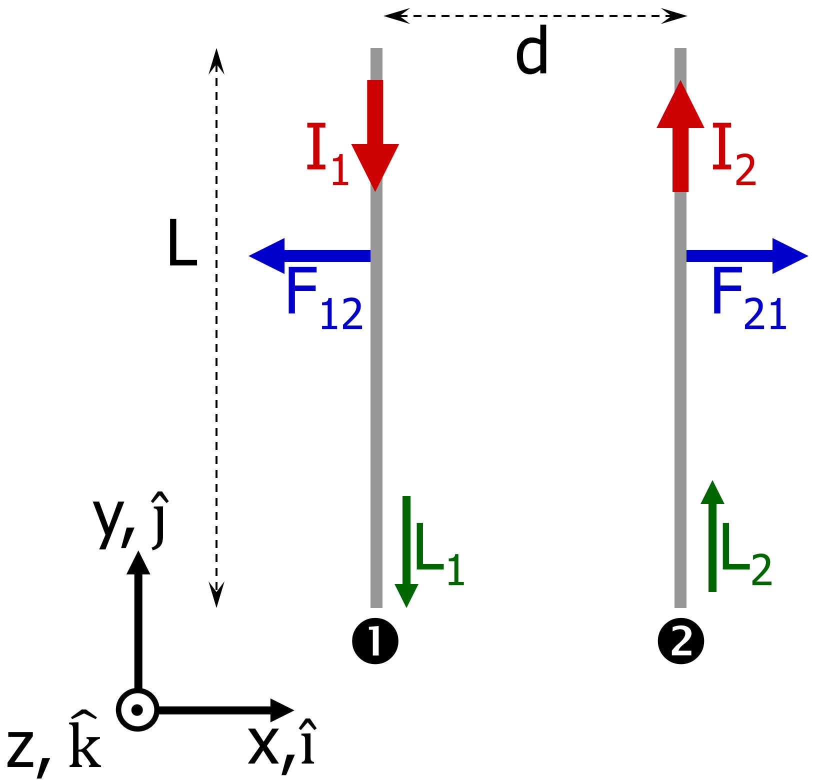
$$\vec{F}_{21} = -\frac{\mu_0 I_1 I_2 L}{2\pi d}\hat{\imath}$$




The force per unit length of wire is $\dfrac{\vec{F}_{21}}{L} = -\dfrac{\mu_0 I_1 I_2}{2\pi d}\hat{\imath}.$

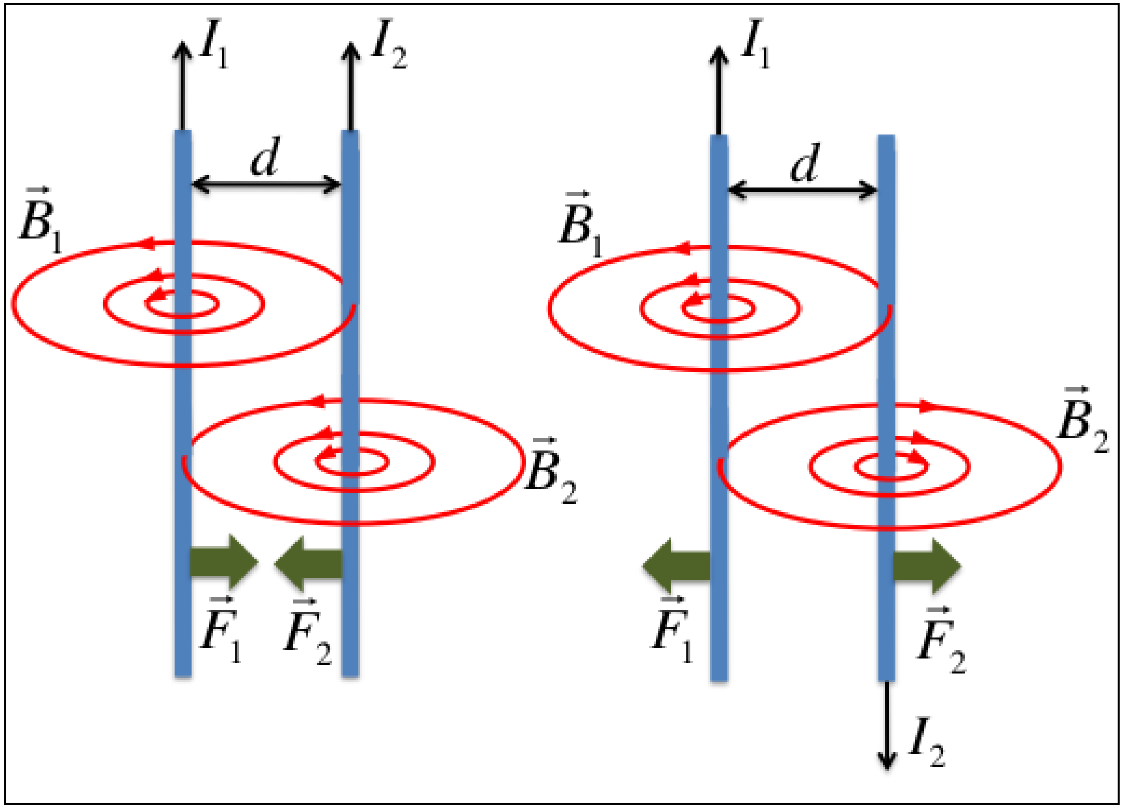
Analogously:
If currents are in opposite directions, force is repulsive.

$$F_{12} = F_{21} = \frac{\mu_0 I_1 I_2 L}{2\pi d}$$




$$F_{12} = F_{21} = \frac{4\pi \times 10^{-7} I_1 I_2 L}{2\pi d}$$

$$= 2 \times 10^{-7} I_1 I_2 \frac{L}{d}$$




Note: experience used to define the Ampere [A]

Official definition of the Ampere:

1 A is the current that produces a force of $2 \times 10^{-7}$ N per meter of length between two long parallel wires placed 1 meter apart in empty space.

**A mechanical measurement can be used to standardize the ampere.**

Today’s agenda:

**Ampere’s Law.**

You must be able to use Ampere’s Law to calculate the magnetic field for high-symmetry current configurations.

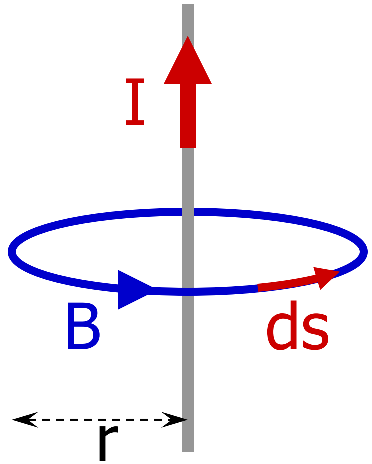
Solenoids.

You must be able to use Ampere’s Law to calculate the magnetic field of solenoids and toroids.

# Recall:

- magnetic field of long straight wire:

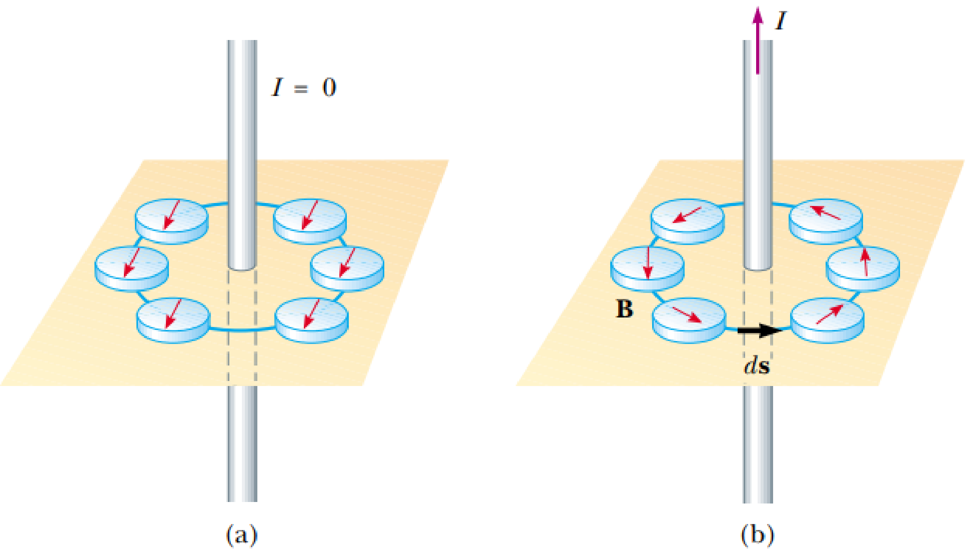
$$B = \frac{\mu_0 I}{2\pi r}$$ winds around the wire




## Line integral of B over a closed circular path around wire:

$$\oint \vec{B} \cdot d\vec{s} = B \oint ds = B(2\pi r)$$

$\vec{B} \parallel d\vec{s}$




$$\oint \vec{B} \cdot d\vec{s} = \left(\frac{\mu_0 I}{2\pi r}\right)(2\pi r) = \mu_0 I$$

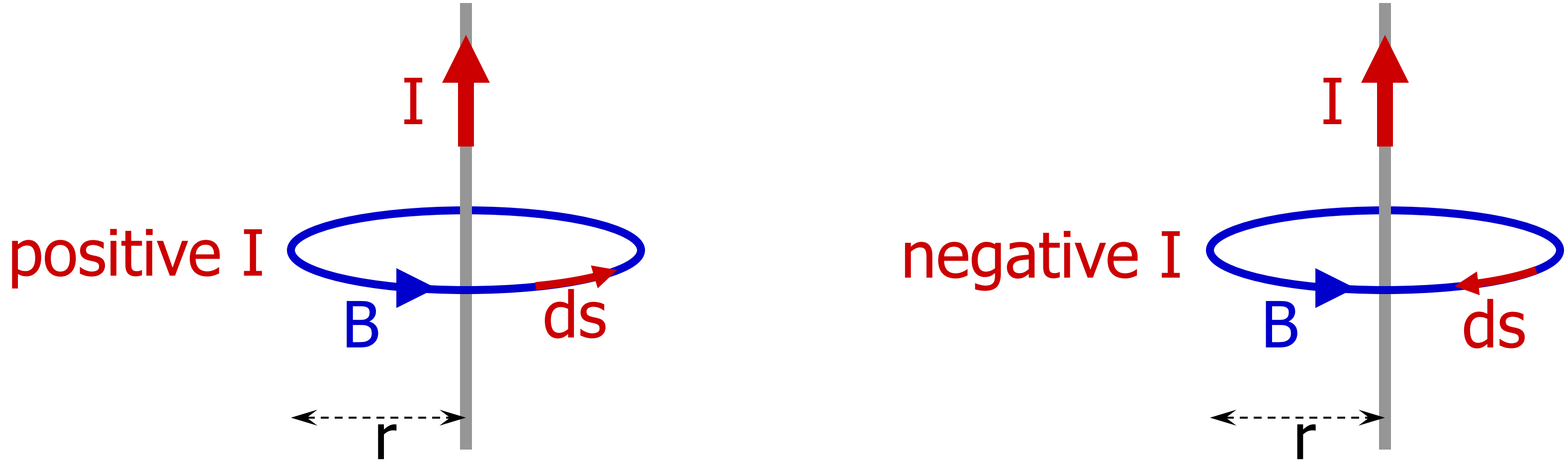
Is this an accident, valid only for this particular situation?

NO! **This is a GENERAL RESULT!**

# Ampere's Law

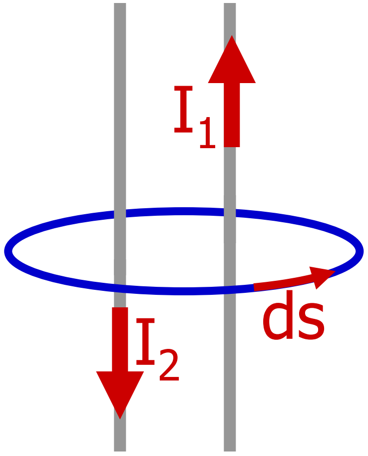
$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \, I_{\mathbf{encl}}$$ Ampere's Law

- $I_{encl}$ is total current that passes through surface bounded by closed path of integration.
- **law of nature:** holds for any closed path and any current distribution
- current I counts positive if integration direction is the same as the direction of $\vec{B}$ from the right hand rule



- if path includes more than one source of current, add all currents (with correct sign).

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \left( I_1 - I_2 \right)$$



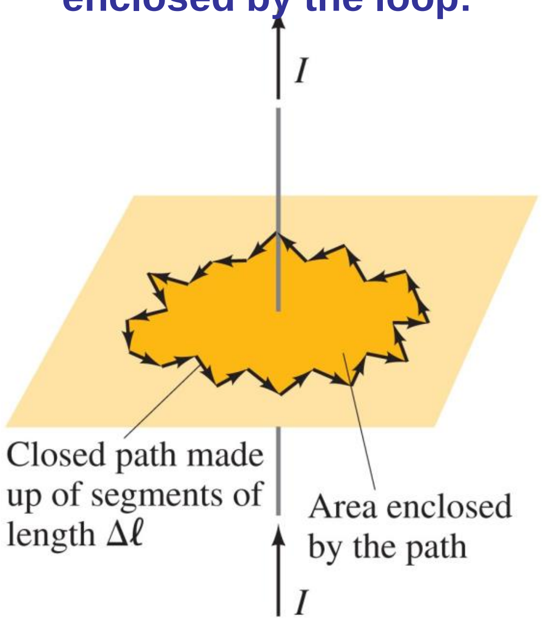

- Ampere's law can be used **to calculate magnetic fields** in **high-symmetry situations**

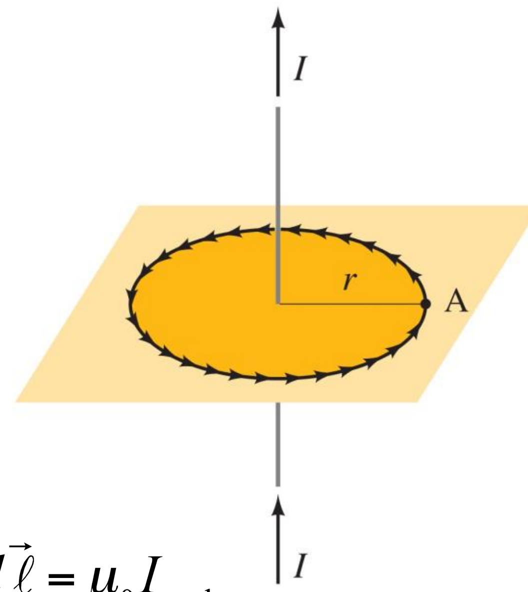
# Ampère's Law

Sometimes the infinitesimal element in the path integral $d\vec{s}$ is also indicated as $d\vec{\ell}$

**Ampère's law relates the magnetic field around a closed loop (Amperian curve) to the total current flowing through the surface enclosed by the loop:**

$$\oint_{loop} \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 \sum I_{\text{encl}}$$




Using Ampère's law to find the field around a long straight wire:

Use a circular path

with the wire at the center;

then ***B*** is tangent

to ***dl*** at every point.

The integral

then gives:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 I_{\text{encl}}$$

$$\oint B\, d\ell = B \oint d\ell = B(2\pi r)$$

$$2\pi r B = \mu_0 I$$

**so $B = \mu_0 I / 2\pi r$**

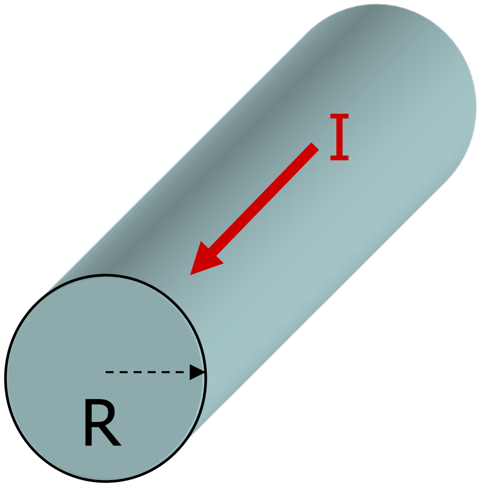
***with direction and orientation given by the right-hand rule***

# Recipe for using Ampere's law to find magnetic fields

- requires **high-symmetry situations** so that line integral can be disentangled
- analogous to Gauss' law calculations for electric field

1. Use symmetry to find direction of magnetic field
2. Choose Amperian path such that
   (a) it respects the symmetry, usually $\vec{B} \,||\, d\vec{s}$
   (b) and goes through point of interest
3. Start from Amperes law, perform integration, solve for B

Example: a cylindrical wire of radius R carries a current I that is uniformly distributed over the wire's cross section. Calculate the magnetic field inside and outside the wire.

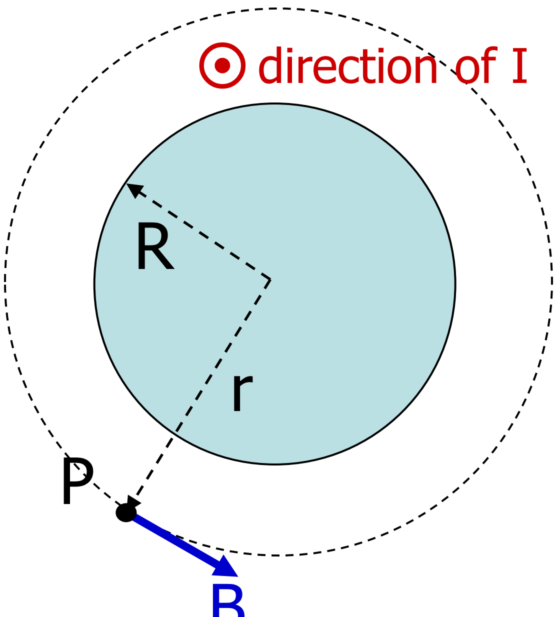


Example: a cylindrical wire of radius R carries a current I that is uniformly distributed over the wire's cross section. Calculate the magnetic field inside and outside the wire.

**Outside the wire:**

1. B field tangential to circles around wire

2. Chose circular Amperian path around wire through P




3. Integrate:

$$\oint \vec{B} \cdot d\vec{s} = B \oint ds = 2\boldsymbol{\pi}\mathbf{r}B = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

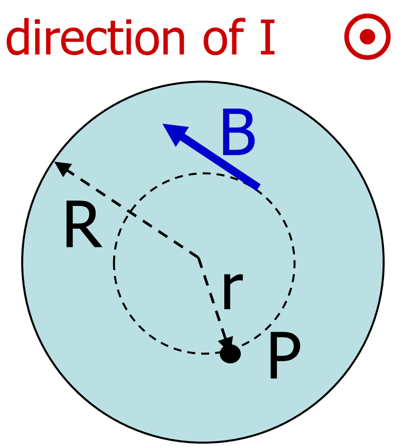
a lot easier than using Biot-Savart Law!

Example: a cylindrical wire of radius R carries a current I that is uniformly distributed over the wire's cross section. Calculate the magnetic field inside and outside the wire.

## Inside the wire:

- Only part of current enclosed by Amperian path

$$I_{encl} = I\frac{(\text{A enclosed by r})}{(\text{A enclosed by R})} = I\frac{(\pi r^2)}{(\pi R^2)} = I\frac{r^2}{R^2}$$



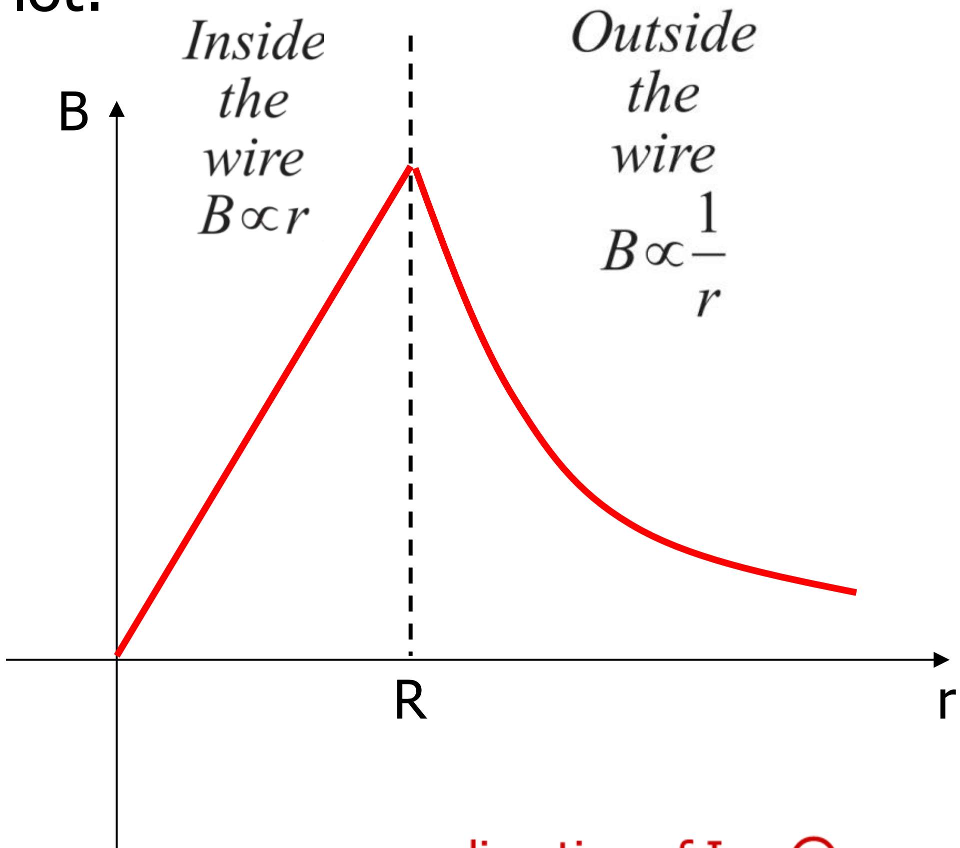

Ampere's law:

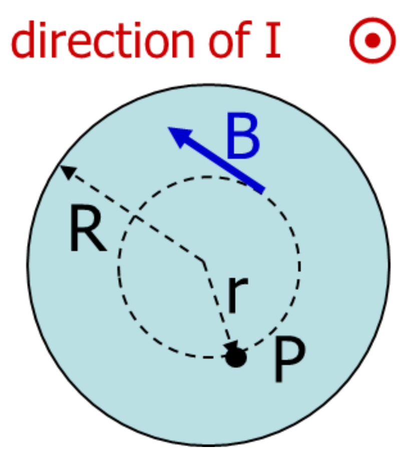
$$\oint \vec{B}\cdot d\vec{s} = B\oint ds = 2\pi r B = \mu_0\, I_{encl} = \mu_0\, I\frac{r^2}{R^2}$$

Solve for B:

$$B = \mu_0\, I\frac{r^2}{2\pi r R^2} = \mu_0\, I\frac{r}{2\pi R^2} = \frac{\mu_0\, I}{2\pi R^2}r$$

Plot:




- inside the wire:

$$B = \frac{\mu_0 i}{2\pi R^2} \cdot r$$

- outside the wire the answer is the same as in the previous exercise:

$$B = \frac{\mu_0 i}{2\pi r}$$

- at $r = R$ both solutions give the same answer

# Calculating Electric and Magnetic Fields

**Electric Field**

in general: Coulomb's Law

for high symmetry configurations: Gauss' Law
(surface integral)

**Magnetic Field**

in general: Biot-Savart Law

for high symmetry configurations: Ampere's Law
(line integral)

This analogy is rather flawed because Ampere's Law is not really the "Gauss' Law of magnetism."

## Today’s agenda:

### Ampere’s Law.

You must be able to use Ampere’s Law to calculate the magnetic field for high-symmetry current configurations.

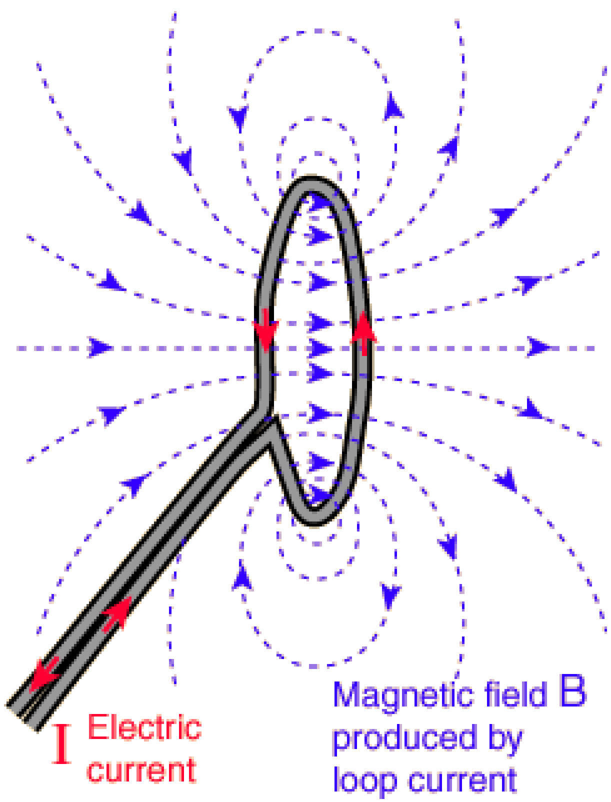
### **Solenoids.**

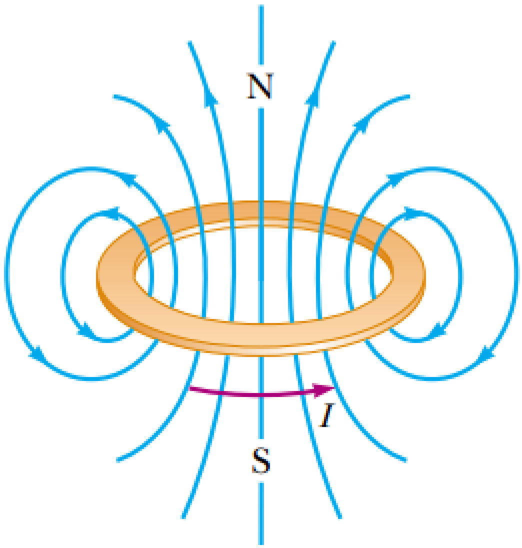
You must be able to use Ampere’s Law to calculate the magnetic field of solenoids and toroids. You must be able to use the magnetic field equations derived with Ampere’s Law to make numerical magnetic field calculations for solenoids and toroids.

# Magnetic Field of a Solenoid

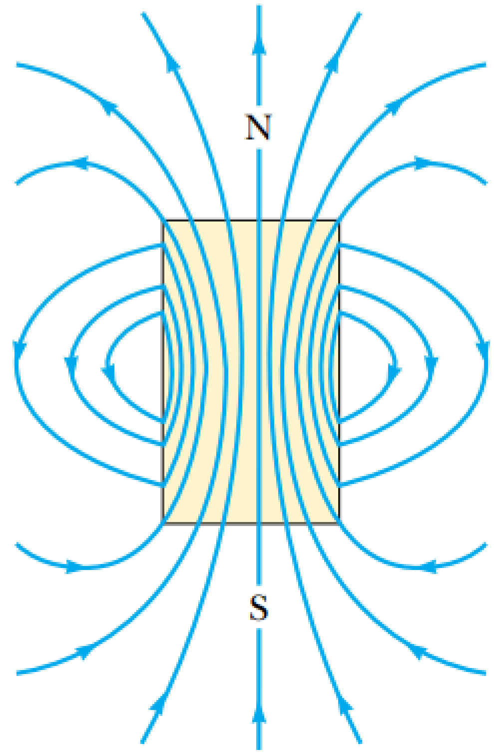
A solenoid is made of many loops of wire, packed closely to form long cylinder.

Single loop:



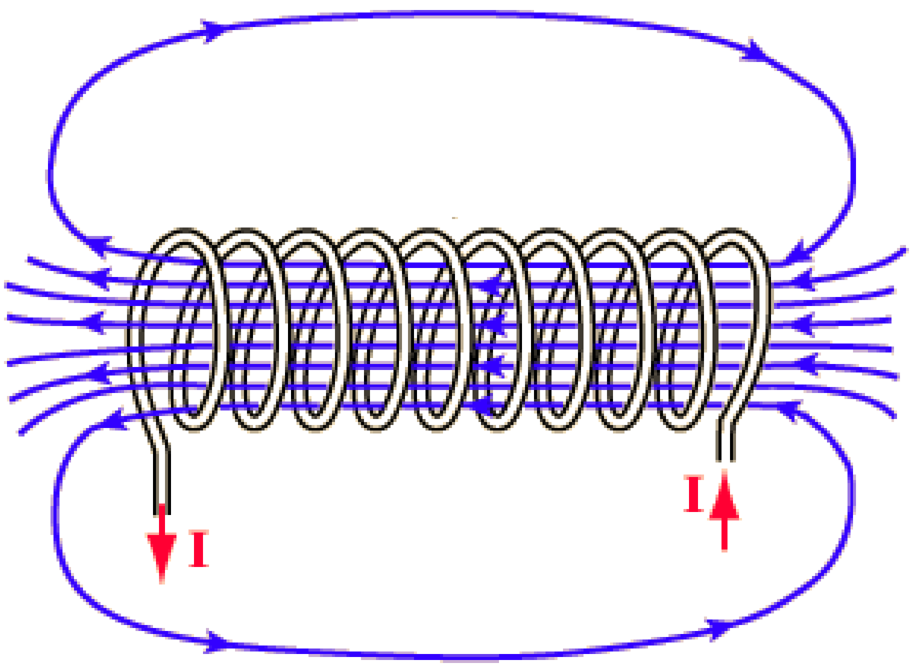
(a)



(c)

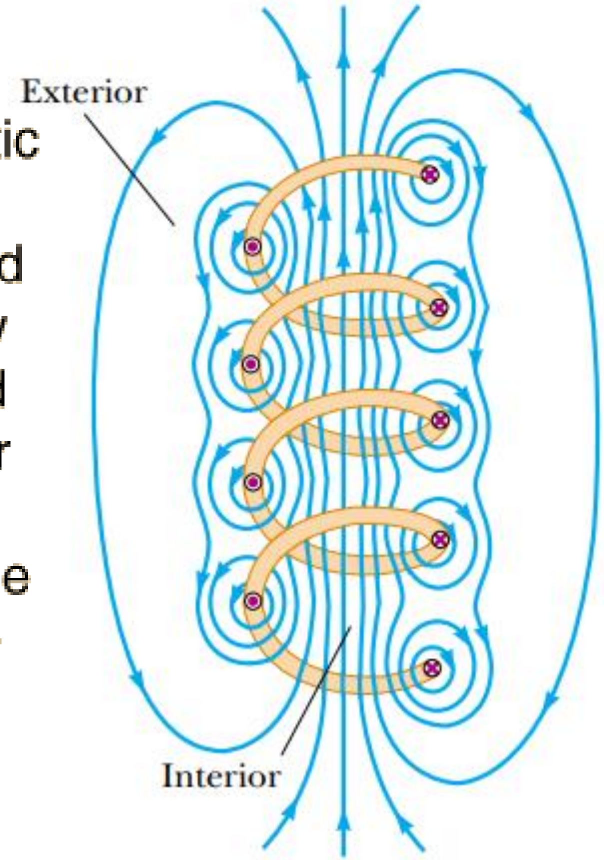
Note the similarity between magnetic field lines surrounding a current loop and that of a bar magnet.

# Stack many loops to make a solenoid:

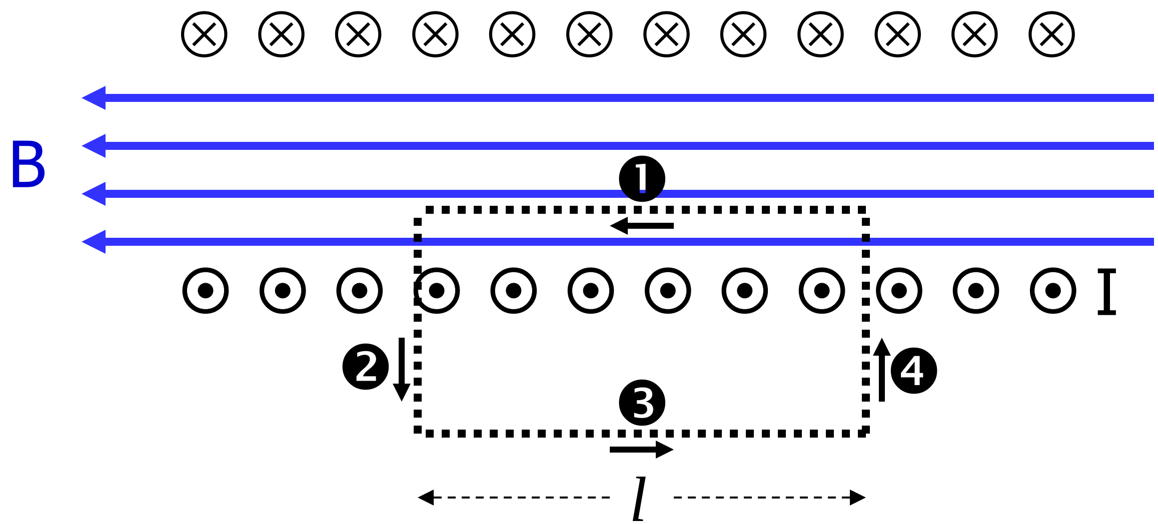



Ought to remind you of the magnetic field of a bar magnet.

The magnetic field is concentrated into a nearly uniform field in the center of a long solenoid. The field outside is weak and divergent.




The magnetic field lines for a loosely wound solenoid.



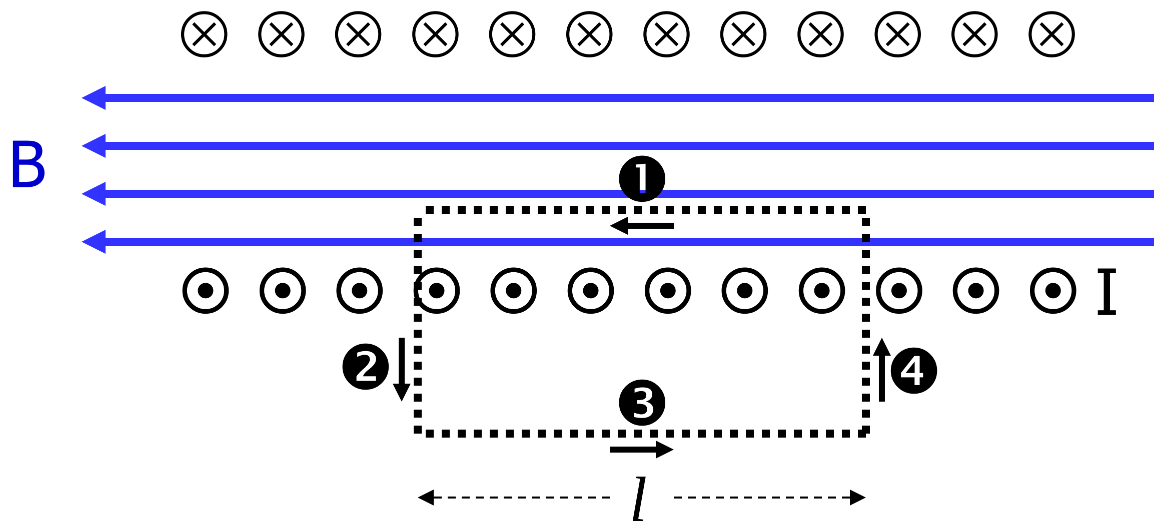

Use Ampere's law to calculate the magnetic field of a solenoid:

$$\oint \vec{B}\cdot d\vec{s} = \int_1 \vec{B}\cdot d\vec{s} + \int_2 \vec{B}\cdot d\vec{s} + \int_3 \vec{B}\cdot d\vec{s} + \int_4 \vec{B}\cdot d\vec{s}$$

$$\oint \vec{B}\cdot d\vec{s} = \quad B\ell \; + \; 0 \; + \; 0 \; + \; 0 \; = \; \mu_0 I_{enclosed}$$

$$B\ell = \mu_0 N I$$

N is the number of loops enclosed by our surface.



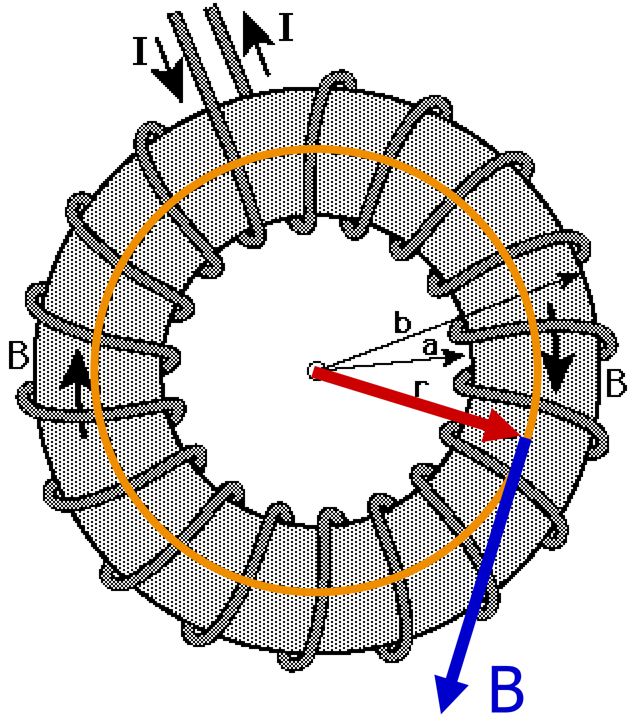

$$B = \mu_0 \frac{N}{\ell} I$$

$$B = \mu_0 \, n \, I$$

Magnetic field of a solenoid of length $l$ , N loops, current I. $n=N/l$ (number of turns per unit length).

The magnetic field inside a long solenoid does not depend on the position inside the solenoid (if end effects are neglected).

# A toroid* is just a solenoid "hooked up" to itself.



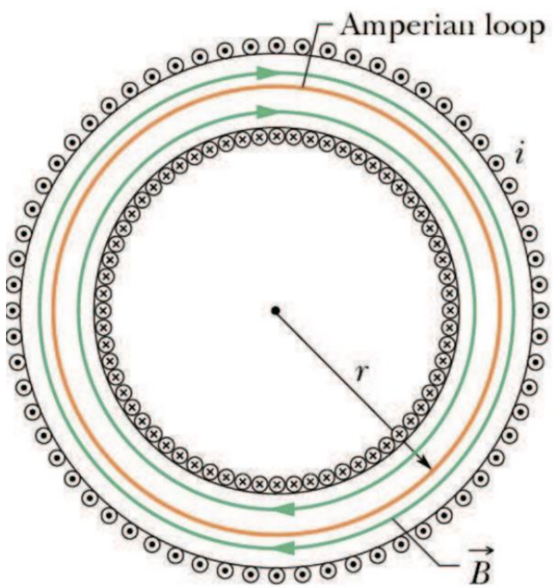

$$\oint \vec{B}\cdot d\vec{s} = \mu_0\, I_{enclosed} = \mu_0\, N\, I$$

$$\oint \vec{B}\cdot d\vec{s} = B\int ds = B(2\pi r)$$

$$B(2\pi r) = \mu_0\, N\, I$$

$$\boxed{B = \frac{\mu_0\, N\, I}{2\pi r}}$$

Magnetic field inside a toroid of N loops, current I.




The magnetic field inside a toroid is not subject to end effects, but is not constant inside (because it depends on r).

Some texts call this also as "toroidal solenoid."

Example: a thin 10-cm long solenoid has a total of 400 turns of wire and carries a current of 2 A. Calculate the magnetic field inside near the center.

$$B = \mu_0 \frac{N}{\ell} I$$

$$B = \left(4\pi \times 10^{-7} \frac{T \cdot m}{A}\right) \frac{(400)}{(0.1\ m)} (2\ A)$$

$$\boxed{B = 0.01\ T}$$

## “Help! Too many similar starting equations!”

$$B = \frac{\mu_0 I}{2\pi r}$$

long straight wire

use Ampere’s law (or note the lack of N)

$$B = \mu_0 \frac{N}{\ell} I$$

solenoid, length l, N turns

field inside a solenoid is constant

$$B = \mu_0 n I$$

solenoid, n turns per unit length

field inside a solenoid is constant

$$B = \frac{\mu_0 N I}{2\pi r}$$

toroid, N loops

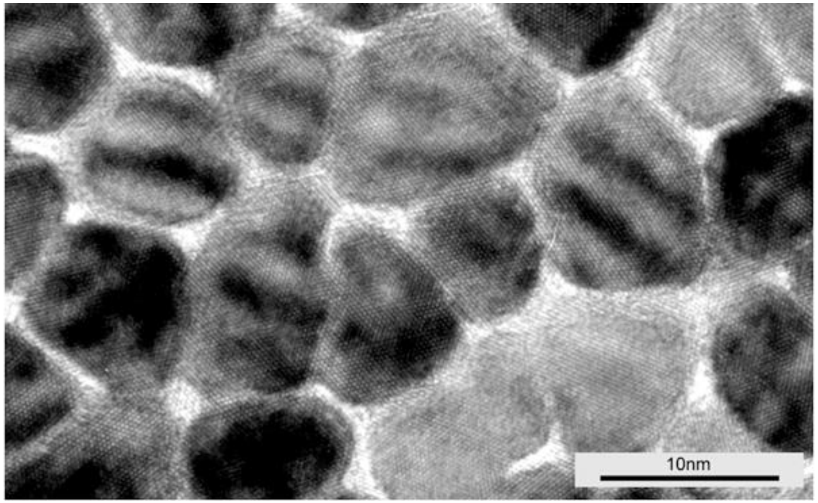
field inside a toroid depends on position (r)

**You can easily derive them using Ampere’s Law**

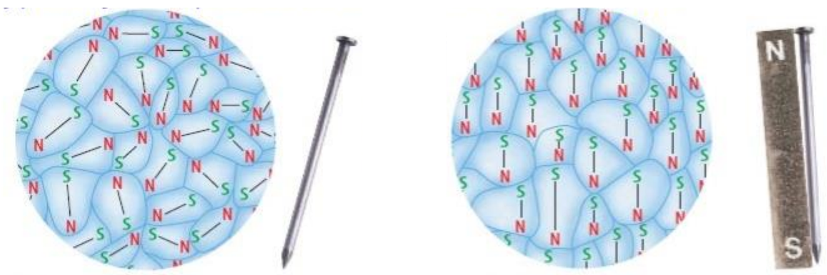
# Magnetic Materials – Ferromagnetism

**Ferromagnetic materials are those that can become strongly magnetized, (iron, nickel)**

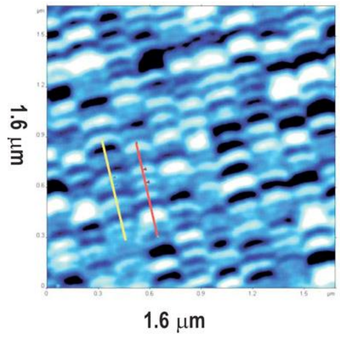
**These materials are made up of tiny crystalloid regions called domains (less than 100 nm in size); the magnetic field in each domain is in a single direction.**




**When the material is non-magnetized, the domains are randomly oriented. They can be partially or fully aligned by placing the material in an external magnetic field.**



**Computer hard drive domains**



# Magnetism in Matter

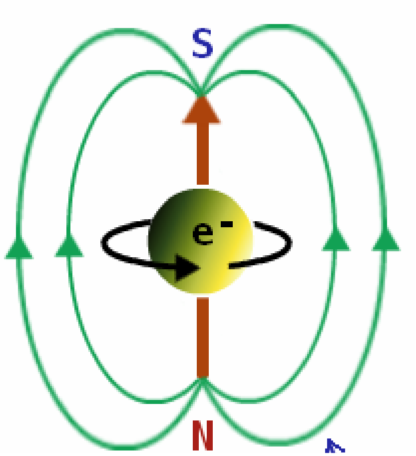
## Origin of Ferromagnetism

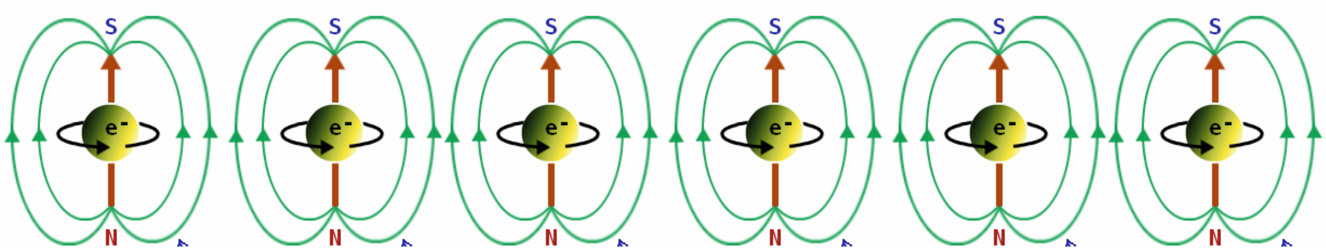
**F.M. originates from quantum nature of electron motion**

**Electron is a rotating charge => current => Magnetic Dipole (spin)**

**In some atoms the outer electrons are aligned parallel => MFs add => MF of atom**

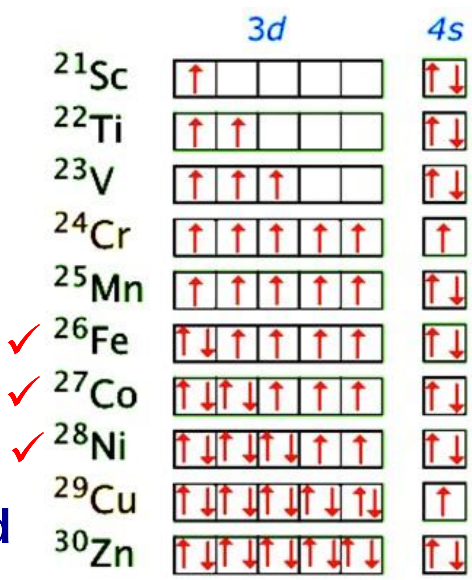
**In some materials atoms within a domain tend to align too (QM effect) => MFs add making MF of a domain:**



## Origin of Ferromagnetism

- **The atoms of Fe, Co and Ni (and rare earths) are little magnets: in the incompletely filled shell of electrons, the electron spins line up—and electrons are themselves magnets.**
- **Only some of the open shell atoms and in specific crystal forms energetically prefer alignment with their neighbors.**
- **All this alignments are fully explained by quantum mechanics (and cannot be explained otherwise).**



EPFL

# Magnetism in Matter

## Paramagnetism and Diamagnetism

Molecules of paramagnetic materials have a small intrinsic magnetic dipole moment, and they tend to align somewhat with an external magnetic field, increasing it.

Molecules of diamagnetic materials have no intrinsic magnetic dipole moment; an external field induces a small dipole moment, but in such a way that the total field is slightly decreased.

**Paramagnetic $\mu \geq \mu_0$** **Diamagnetic $\mu \leq \mu_0$**

Characterized by magnetic susceptibility $\chi_m = \mu/\mu_0 - 1.$

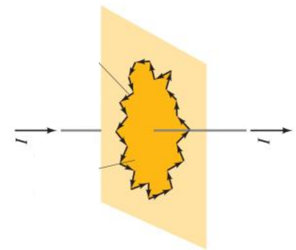
**TABLE 28–1 Paramagnetism and Diamagnetism: Magnetic Susceptibilities**

| Paramagnetic substance | $\chi_m$ | Diamagnetic substance | $\chi_m$ |
|---|---|---|---|
| Aluminum | $2.3 \times 10^{-5}$ | Copper | $-9.8 \times 10^{-6}$ |
| Calcium | $1.9 \times 10^{-5}$ | Diamond | $-2.2 \times 10^{-5}$ |
| Magnesium | $1.2 \times 10^{-5}$ | Gold | $-3.6 \times 10^{-5}$ |
| Oxygen (STP) | $2.1 \times 10^{-6}$ | Lead | $-1.7 \times 10^{-5}$ |
| Platinum | $2.9 \times 10^{-4}$ | Nitrogen (STP) | $-5.0 \times 10^{-9}$ |
| Tungsten | $6.8 \times 10^{-5}$ | Silicon | $-4.2 \times 10^{-6}$ |

# Summary of Lecture 8

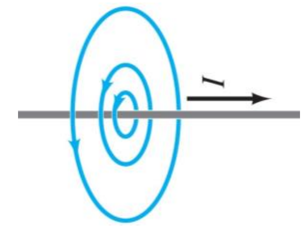
- **Current creates magnetic field: (Amper's law)**

$$\oint_{loop} \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 \sum I_{\text{encl}}$$



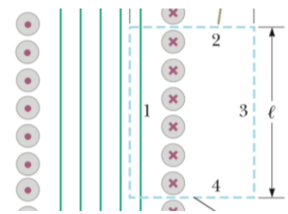
- **Magnetic field of a long, straight current-carrying wire:**

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$




- **Magnetic field inside a solenoid:**

$$B = \mu_0 n I, \quad n = \frac{N}{\ell}$$




- **Strong permanent magnets are made of ferromagnetic materials with $\mu >> \mu_0$:**